OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS 4763

Mechanics 3

Monday **22 MAY 2006** Morning 1 hour 30 minutes

Additional materials:
8 page answer booklet
Graph paper
MEI Examination Formulae and Tables (MF2)

TIME 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g\ \mathrm{m\,s^{-2}}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is 72.

This question paper consists of 5 printed pages and 3 blank pages.

1 **(a)** **(i)** Find the dimensions of power. [3]

In a particle accelerator operating at power P, a charged sphere of radius r and density ρ has its speed increased from u to $2u$ over a distance x. A student derives the formula

$$x = \frac{28\pi r^3 u^2 \rho}{9P}.$$

(ii) Show that this formula is not dimensionally consistent. [5]

(iii) Given that there is only one error in this formula for x, obtain the correct formula. [3]

(b) A light elastic string, with natural length 1.6 m and stiffness $150\,\mathrm{N\,m^{-1}}$, is stretched between fixed points A and B which are 2.4 m apart on a smooth horizontal surface.

(i) Find the energy stored in the string. [2]

A particle is attached to the mid-point of the string. The particle is given a horizontal velocity of $10\,\mathrm{m\,s^{-1}}$ perpendicular to AB (see Fig. 1.1), and it comes instantaneously to rest after travelling a distance of 0.9 m (see Fig. 1.2).

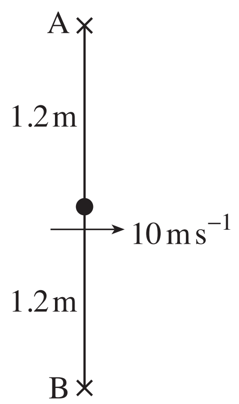

Fig. 1.1

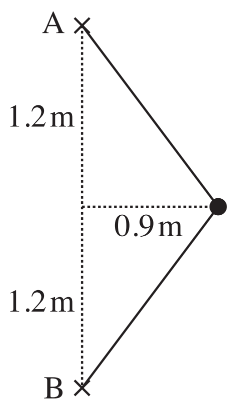

Fig. 1.2

(ii) Find the mass of the particle. [5]

2 **(a)** A particle P of mass 0.6 kg is connected to a fixed point by a light inextensible string of length 2.8 m. The particle P moves in a horizontal circle as a conical pendulum, with the string making a constant angle of 55° with the vertical.

(i) Find the tension in the string. [2]

(ii) Find the speed of P. [4]

(b) A turntable has a rough horizontal surface, and it can rotate about a vertical axis through its centre O. While the turntable is stationary, a small object Q of mass 0.5 kg is placed on the turntable at a distance of 1.4 m from O. The turntable then begins to rotate, with a constant angular acceleration of 1.12 rad s^{-2}. Let ω rad s^{-1} be the angular speed of the turntable.

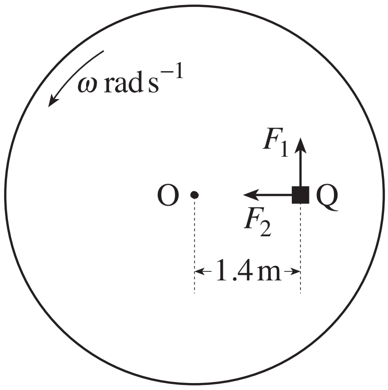

Fig. 2

(i) Given that Q does not slip, find the components F_1 and F_2 of the frictional force acting on Q perpendicular and parallel to QO (see Fig. 2). Give your answers in terms of ω where appropriate. [4]

The coefficient of friction between Q and the turntable is 0.65.

(ii) Find the value of ω when Q is about to slip. [5]

(iii) Find the angle which the frictional force makes with QO when Q is about to slip. [3]

[Turn over

3 A fixed point A is 12 m vertically above a fixed point B. A light elastic string, with natural length 3 m and modulus of elasticity 1323 N, has one end attached to A and the other end attached to a particle P of mass 15 kg. Another light elastic string, with natural length 4.5 m and modulus of elasticity 1323 N, has one end attached to B and the other end attached to P.

(i) Verify that, in the equilibrium position, AP = 5 m. [3]

The particle P now moves vertically, with both strings AP and BP remaining taut throughout the motion. The displacement of P above the equilibrium position is denoted by x m (see Fig. 3).

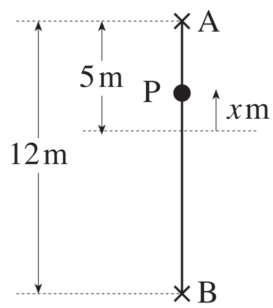

Fig. 3

(ii) Show that the tension in the string AP is $441(2-x)$ N and find the tension in the string BP. [3]

(iii) Show that the motion of P is simple harmonic, and state the period. [4]

The minimum length of AP during the motion is 3.5 m.

(iv) Find the maximum length of AP. [1]

(v) Find the speed of P when AP = 4.1 m. [3]

(vi) Find the time taken for AP to increase from 3.5 m to 4.5 m. [4]

4 The region bounded by the curve $y = \sqrt{x}$, the x-axis and the lines $x = 1$ and $x = 4$ is rotated through 2π radians about the x-axis to form a uniform solid of revolution.

(i) Find the x-coordinate of the centre of mass of this solid. [6]

From this solid, the cylinder with radius 1 and length 3 with its axis along the x-axis (from $x = 1$ to $x = 4$) is removed.

(ii) Show that the centre of mass of the remaining object, Q, has x-coordinate 3. [5]

This object Q has weight 96 N and it is supported, with its axis of symmetry horizontal, by a string passing through the cylindrical hole and attached to fixed points A and B (see Fig. 4). AB is horizontal and the sections of the string attached to A and B are vertical. There is sufficient friction to prevent slipping.

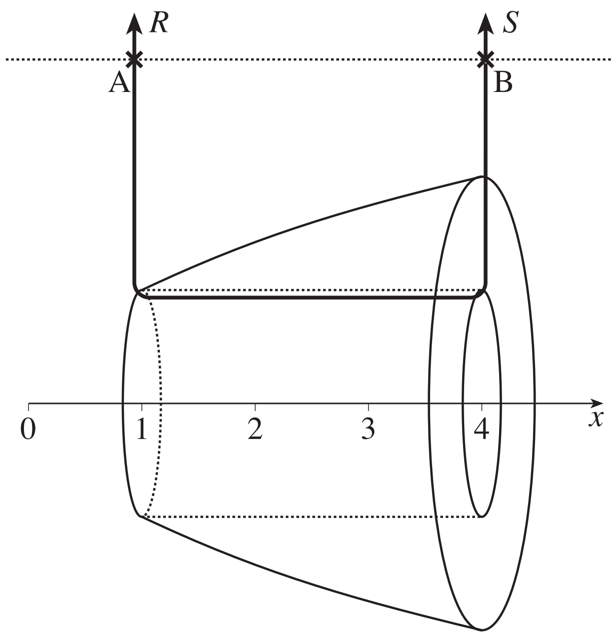

Fig. 4

(iii) Find the support forces, R and S, acting on the string at A and B

(A) when the string is light, [4]

(B) when the string is heavy and uniform with a total weight of 6 N. [3]

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1(a)(i)	[Force] $= \mathrm{M\,L\,T^{-2}}$ [Power] = [Force] $\times$ [Distance] $\div$ [Time] = [Force] $\times \mathrm{L\,T^{-1}}$ $= \mathrm{M\,L^2\,T^{-3}}$	B1 M1 A1 **3**	or [Energy] $= \mathrm{M\,L^2\,T^{-2}}$ or [Energy] $\times \mathrm{T^{-1}}$
(ii)	$[\mathrm{RHS}] = \dfrac{(\mathrm{L})^3(\mathrm{L\,T^{-1}})^2(\mathrm{M\,L^{-3}})}{\mathrm{M\,L^2\,T^{-3}}}$ $= \mathrm{T}$ [LHS] = L so equation is not consistent	B1B1 M1 A1 E1 **5**	For $(\mathrm{L\,T^{-1}})^2$ and $(\mathrm{M\,L^{-3}})$ Simplifying dimensions of RHS With all working correct (cao) *SR* '$\ldots \mathrm{L} = \frac{28}{9}\pi\,\mathrm{T}$, so inconsistent ' can earn B1B1M1A1E0
(iii)	[RHS] needs to be multiplied by $\mathrm{L\,T^{-1}}$ which are the dimensions of *u* Correct formula is $x = \dfrac{28\pi r^3 u^3 \rho}{9P}$	M1 A1 A1 cao **3**	RHS must appear correctly
	OR $x = k\,r^{\alpha}\,u^{\beta}\,\rho^{\gamma}\,P^{\delta}$ M1 $\beta = 3$ A1 $x = \dfrac{28\pi r^3 u^3 \rho}{9P}$ A1		Equating powers of one dimension
(b)(i)	Elastic energy is $\frac{1}{2}\times 150\times 0.8^2$ $= 48\ \mathrm{J}$	M1 A1 **2**	*Treat use of modulus* $\lambda = 150\ \mathrm{N}$ *as MR*
(ii)	In extreme position, length of string is $2\sqrt{1.2^2+0.9^2}$ $(=3)$ elastic energy is $\frac{1}{2}\times 150\times 1.4^2$ $(=147)$ By conservation of energy, $147 - 48 = \frac{1}{2}\times m\times 10^2$ Mass is 1.98 kg	B1 M1 M1 A1 A1 **5**	for $\sqrt{1.2^2+0.9^2}$ or 1.5 or 3 allow M1 for $(2\times)\frac{1}{2}\times 150\times 0.7^2$ Equation involving EE and KE

2 (a)(i)	Vertically, $T\cos 55° = 0.6\times 9.8$ Tension is 10.25 N	M1 A1 **2**	
(ii)	Radius of circle is $r = 2.8\sin 55° \quad (=2.294)$	B1	
	Towards centre, $T\sin 55° = 0.6\times\dfrac{v^2}{2.8\sin 55°}$	M2	Give M1 for one error
	OR $T\sin 55° = 0.6\times(2.8\sin 55°)\times\omega^2$ M1 $\omega = 2.47$ $v = (2.8\sin 55°)\,\omega$ M1		or $T = 0.6\times 2.8\times\omega^2$ *Dependent on previous M1*
	Speed is 5.67 m s^{-1}	A1 **4**	
(b)(i)	Tangential acceleration is $r\alpha = 1.4\times 1.12$ $F_1 = 0.5\times 1.4\times 1.12$ $= 0.784\text{ N}$ Radial acceleration is $r\omega^2 = 1.4\omega^2$ $F_2 = 0.5\times 1.4\omega^2$ $= 0.7\omega^2\text{ N}$	M1 A1 M1 A1 **4**	*SR* $F_1 = -0.784,\ F_2 = -0.7\omega^2$ *penalise once only*
(ii)	Friction $F = \sqrt{F_1^{\,2} + F_2^{\,2}}$ Normal reaction $R = 0.5\times 9.8$ About to slip when $F = \mu\times 0.5\times 9.8$ $\sqrt{0.784^2 + 0.49\omega^4} = 0.65\times 0.5\times 9.8$ $\omega = 2.1$	M1 M1 A1 A1 A1 cao **5**	For LHS and RHS *Both dependent on M1M1*
(iii)	$\tan\theta = \dfrac{F_1}{F_2}$ $= \dfrac{0.784}{0.7\times 2.1^2}$ Angle is $14.25°$	M1 A1 A1 **3**	Allow M1 for $\tan\theta = \dfrac{F_2}{F_1}$ etc Accept 0.249 rad

3 (i)	$T_{AP} = \frac{1323}{3} \times 2 \quad (= 882)$ $T_{BP} = \frac{1323}{4.5} \times 2.5 \quad (= 735)$ $T_{AP} - mg - T_{BP} = 882 - 15 \times 9.8 - 735 = 0$ so P is in equilibrium	B1 B1 E1 **3**	
	OR $\frac{1323}{3}(AP - 3) = \frac{1323}{4.5}(BP - 4.5) + 15 \times 9.8$ B2 $AP + BP = 12$ and solving, $AP = 5$ E1		Give B1 for one tension correct
(ii)	Extension of AP is $5 - x - 3 = 2 - x$ $T_{AP} = \frac{1323}{3}(2 - x) = 441(2 - x)$ Extension of BP is $7 + x - 4.5 = 2.5 + x$ $T_{BP} = \frac{1323}{4.5}(2.5 + x) = 294(2.5 + x)$	 E1 B1 B1 **3**	
(iii)	$441(2 - x) - 15 \times 9.8 - 294(2.5 + x) = 15\frac{d^2x}{dt^2}$ $\frac{d^2x}{dt^2} = -49x$ Motion is SHM with period $\frac{2\pi}{\omega} = \frac{2\pi}{7} = 0.898 \text{ s}$	M1 A1 M1 A1 **4**	Equation of motion involving 3 forces Obtaining $\frac{d^2x}{dt^2} = -\omega^2 x \ (+c)$ Accept $\frac{2}{7}\pi$
(iv)	Centre of motion is AP = 5 If minimum value of AP is 3.5, amplitude is 1.5 Maximum value of AP is 6.5 m	B1 **1**	
(v)	When $AP = 4.1, \quad x = 0.9$ Using $v^2 = \omega^2(A^2 - x^2)$ $v^2 = 49(1.5^2 - 0.9^2)$ Speed is 8.4 m s^{-1}	 M1 A1 A1 **3**	Accept ± 8.4 or -8.4
	OR $x = 1.5\sin 7t$ When $x = 0.9, \ 7t = 0.6435 \quad (t = 0.0919)$ $v = 7 \times 1.5\cos 7t$ M1 $= 10.5\cos(0.6435)$ A1 $= 8.4$ A1		or $x = 1.5\cos 7t$ or $7t = 0.9273 \quad (t = 0.1325)$ or $v = -7 \times 1.5\sin 7t$ $= (-)\ 10.5\sin(0.9273)$

(vi)	$x = 1.5\cos 7t$	M1 A1	For $\cos(\sqrt{49}\,t)$ or $\sin(\sqrt{49}\,t)$ or $x = 1.5\sin 7t$ *M1A1 above can be awarded in (v) if not earned in (vi)*
	When $1.5\cos 7t = 0.5$	M1	or other fully correct method to find the required time e.g. $0.400 - 0.224$ or $0.224 - 0.049$
	Time taken is 0.176 s	A1 **4**	Accept 0.17 or 0.18

4 (i)	$\int \pi y^2\,dx = \int_1^4 \pi x\,dx$ $= \left[\frac{1}{2}\pi x^2\right]_1^4 = 7.5\pi$ $\int \pi x y^2\,dx$ $= \int_1^4 \pi x^2\,dx \quad = \left[\frac{1}{3}\pi x^3\right]_1^4 \quad (= 21\pi)$ $\bar{x} = \frac{21\pi}{7.5\pi}$ $= 2.8$	M1 A1 M1 A1 M1 A1 **6**	*π may be omitted throughout*
(ii)	Cylinder has mass $3\pi\rho$ Cylinder has CM at $x = 2.5$ $(4.5\pi\rho)\bar{x} + (3\pi\rho)(2.5) = (7.5\pi\rho)(2.8)$ $\bar{x} = 3$	B1 B1 M1 A1 E1 **5**	Or volume 3π Relating three CMs *(ρ and / or π may be omitted)* or equivalent, e.g. $\bar{x} = \frac{(7.5\pi\rho)(2.8) - (3\pi\rho)(2.5)}{7.5\pi\rho - 3\pi\rho}$ Correctly obtained
(iii)(*A*)	Moments about A, $S \times 3 - 96 \times 2 = 0$ $S = 64$ N Vertically, $R + S = 96$ $R = 32$ N	M1 A1 M1 A1 **4**	Moments equation or another moments equation *Dependent on previous M1*
(*B*)	Moments about A, $S \times 3 - 96 \times 2 - 6 \times 1.5 = 0$ Vertically, $R + S = 96 + 6$ $R = 35$ N, $S = 67$ N	M1 A1 A1 **3**	Moments equation Both correct
	OR Add 3 N to each of *R* and *S* M1 $R = 35$ N, $S = 67$ N A2		*Provided $R \neq S$* Both correct

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General Comments

The standard of work on this paper was very good indeed, with about half the candidates scoring 60 marks or more (out of 72), and only about 10% of the candidates scoring less than 30 marks. The only topics which caused widespread difficulty were radial and transverse components of circular motion in Q.2(b) and some aspects of simple harmonic motion in Q.3.

Comments on Individual Questions

1) **Dimensional Analysis and Elastic Energy**
The average mark on this question was about 15 (out of 18).
The dimensional work in part (a) was well understood. The most common error was getting the dimensions of power wrong; this was ‘followed through’ and most of the following marks were usually earned.
The application of elastic energy in part (b) was also well done, the main errors being slips in finding the extension of the string in the extreme position, and forgetting about the initial elastic energy (despite this having been found in part (ii)). Some candidates treated the stiffness as if it were the modulus of elasticity.

2) **Circular Motion**
This was the worst answered question, with an average mark of about 11.
For the conical pendulum in part (a), many candidates tried to find the tension by resolving along the string ($T = mg\cos 55°$) instead of vertically ($T\cos 55° = mg$), and the radius was often taken to be 2.8 instead of $2.8\sin 55°$.
In part (b)(i) the radial component F_2 was usually given correctly, but only about half the candidates gave the transverse component $F_1 = mr\alpha$ correctly; the mass or the radius was very often omitted. Some thought that F_1 was vertical and wrote $F_1 = mg$. Correct solutions to part (b)(ii) were rare; the majority wrote $F_2 = \mu mg$ instead of $\sqrt{F_1^{\,2} + F_2^{\,2}} = \mu mg$. The final part (b)(iii) was well understood.

3) **Elasticity and Simple Harmonic Motion**
The average mark on this question was about 14.
Parts (i), (ii) and (iv) were answered correctly by the great majority of candidates.
In part (iii), most candidates were able to obtain the correct equation of motion, although sign errors were quite common, as was omission of the weight.
In part (v), most candidates knew that they should apply $v^2 = \omega^2(A^2 - x^2)$, but not all could find the appropriate values for *A* and *x*.
In part (vi), most candidates used $x = A\sin\omega t$, rather than the more efficient $x = A\cos\omega t$, and the correct answer was often obtained. The main difficulties lay in finding appropriate values for *x*.

4) **Centres of Mass**
This was the best answered question, with an average mark of about 15.
In part (i) the centre of mass of the solid of revolution was found confidently and correctly by almost every candidate.
In part (ii), the principles were well understood, and most candidates managed to obtain the given answer, sometimes after one or two false starts.
In part (iii)(*A*), most candidates knew what to do and did it correctly; a fairly common error was to take the length of the object to be 4 instead of 3. Most candidates could also cope with the refinement in part (iii)(*B*); the majority took moments again, but some did realise that they could just add 3N to each of the previous values.